Name————————————, ID ————————

MTH 203, CALCULUS III , Exam One

Ayman Badawi

QUESTION 1. a) Find a parametric equations of the line that is perpendicular to the plane $3x - 2y + 4z = 6$ and intersects the plane at the point $(0, 1, 2)$

b) Find an equation of the plane P where each point in P is equidistant from the two points $(1, -4, 5)$ and $(3, -2, 1)$.

c) Find an equation of the plane P that contains the line $< t - 1, 2t + 3, 5 >$ and the point $(4, 2, -1)$. Does P contain the vector $< 10, -2, -12 >$? explain

QUESTION 2. a) Given a, b, c, d are some constants where $< at, bt, b >$ is the line of intersection of the two planes $cx + dy + dz = -4$ and $2x - 3y + 2z = 8$. Find the values of a, b, c, d.

b) Given that the two planes, $P_1 : 2x + y + z = 1$ and $P_2 : -2x + y + 3z = -4$ intersects in a line L.
i) Find a parametric equations of L.

ii) Find the distance between the point $(2, 2, 0)$ and P_1.

iii) Find the distance between $(2, 2, 0)$ and L

QUESTION 3. a) The two objects $4x^2 + 9y^2 = 1$ and $z = xy + 2$ intersects in a curve (vector function) $r(t)$. Find a parametric equations of $r(t)$.

b) Let $r(t) =< 2t + 1, \dfrac{e^t - ln(t+1) - 1}{cos(t) - 1}, t + 1 >$

i) Find the domain of $r(t)$.

ii) Find $lim_{t \to 0} \quad r(t)$

c) Find the arc-length of $r(t) =< 2e^t, 3e^{-t}, \sqrt{12}t >$ when t is between 0 and $ln(0.5)$.

QUESTION 4. a)Find the area of the triangle that has the vertices $(1,2),(1,4),(0,2)$.

b) Given that the three vectors (having the same initial point), $V =< 2,2,0 >, W =< 1,-2.0 >$, and $D =< 1,1,-2 >$ do not lie in the same plane. However, they form a parallel-pipe (let call it a twisted-cube). Find the volume of the twisted-cube.

c) Find the equations of the tangent line and the normal line to the curve $r(t) =< sin(2t), 2cos(2t), \sqrt{3}sin(2t) - 2\sqrt{3} >$ at the point that is determined by letting $t = \pi/4$

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MTH 203, Calculus III, EXAM II

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QUESTION 1. (i) Does $lim_{\ (x,y)\to(0,0)} \ \frac{4xy^2}{x^2+y^2}$ exist? if yes then find it

(ii) Find the partial derivative dz/dx for $e^z = xyz + yz^2 + xln(y) + 3z + 2x$

(iii) Linearalize $f(x,y) = ye^{2xy}$ at $(0,1)$. Then use the linearalization to approximate $f(-0.2, 1.1)$

(iv) Find the equation of the tangent plane to the solid object in 3D determined by $x^2 + 2y + 3zx = 9$ at the point $(1,1,2)$

QUESTION 2. (i) Use the chain rule to find dz/dx, where $z = 3x^2 + xy^2 + 1, x = s + 2t - u, y = stu^2$ when $s = 4, t = 2, u = 1$

(ii) Given $f(x, y) = 2xe^{y-2} + xy$. Find the directional derivative $D_u(f)$ at the point $(1, 2)$ in the direction of the vector $v = 3i + 4j$. In what direction does f have the maximum rate of change at the given point? what is the maximum change?

(iii) Let $f(x, y) = (y^2 + 4)e^{x^2} - 6y + 10$ Find the critical points of $f(x, y)$. Does $f(x, y)$ have local min. (max) values? if yes then find them.

QUESTION 3. (i) Find the volume of the solid object that has a rectangular basis, say D, in the xy-plane where (0, 0), (1, 1), (1, 2) , and (0, 1) are the vertices of D and the height z is a function in terms of x and y where $z = 2x + 2$.

(ii) Find the volume of the solid subject that has a basis consists of all points in the upper half of the xy-plane that are enclosed between the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$, and the height z is given as a function in terms of x and y where $z = 4x^2 + 4y^2$.

(iii) Find the surface area of the solid subject that has a basis consists of all points in the first quadrant of the xy-plane that are enclosed between the two graphs $y = x^2 + 24x$ and $y = x^2$ where $0 \leq x \leq 2$, and the height z is given as a function in terms of x and y where $z = x^2 + 3y$

QUESTION 4. Given the force field $F(x, y) = (1 + 2xy)i + (x^2 - 3y^2)j$

(i) Is $F(x, y)$ conservative? If yes, find a function $g(x, y)$ such that $\nabla g = F(x, y)$

(ii) A particle moves along line segments from (0, 0) to (4, 1) to (3, 4) to (2, 2) (counter clock-wise). Find the work done by the above force $F(x, y)$ in moving the particle along the given line segments from (0, 0) to (2, 2)

(iii) Let C be the part of the curve of the ellipse $x^2 + y^2/4 = 1$ in the second quadrant of the xy-plane, and assume that C is positively oriented. Find the area of the side that is bounded between the function $z = -9xy$ and C.

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MTH 203, Calculus III, Final EXAM Fall 2013

Ayman Badawi

QUESTION 1. a) Find a vector in the same direction as < -2, 4, 2 > but has length 6.

b) Find the angle between the two vectors $U = i + \sqrt{3}j$ and $V = i$.

c) Find the spherical coordinates and the cylindrical coordinates of the point (2, -2, 1)

d) Let $b =< 1, 1, 2 >$ and $a =< 2, 0, 4 >$ find two vectors say u, v such that u is in the direction of a, v is perpendicular to a, and $u + v = b$.

QUESTION 2. Find parametric equations of the line that passes through the point Q = (1, -1, 10) and perpendicular to the plane P : $3x - 4y = 22$. Then find the distance between Q and P.

QUESTION 3. Find equations of two perpendicular planes, say P_1 and P_2, that intersect in the line L : $< 1+2t,\ -1+t,\ 3-t >$. [Hint: There are infinitely many possibilities for P_1 and P_2. You only need to give me one possibility for P_1 and another for P_2].

QUESTION 4. Given $xye^z + z^2 + zx^2 - 8yz - y^2 + c = 0$ for some constant c , $x = 3t + u$, $y = t^2 - u$. Find dz/dt when $t = 1$, $u = -2$ and $z = 0$. Then find the constant c.

QUESTION 5. Find all points at which the direction of the fastest change of the function $f(x, y) = 0.5x^2 - 2x + y^2 - 6y$ is in the direction of $i + 2j$ [HINT: Recall If a vector U is in the direction of V, then $U = cV$ for some constant c]

QUESTION 6. Find an equation of the tangent plane to the surface $x^2 + xy + yz = 3e^{xyz-1}$ at the point $(1, 1, 1)$.

QUESTION 7. If possible find all local minimum and maximum values of $f(x, y) = y^2 - 2ycos(x)$ where $0 < x < 4$ [hint : note that $2\pi > 4$]

QUESTION 8. Given the force field $F(x, y) = yi + 5xj$. A particle moves along line segments from (0, 3) to (1, 3) to (4, 6) to (0, 6), then back to (0, 3) (counter clock-wise). Find the work done by the force $F(x, y)$ in moving the particle along the given line segments from (0, 3) back to (0, 3). [Hint: Recall that if $F = A(x, y)i + B(x, y)j$, then $\int_C F \,.\, dr = \int_C A(x, y)\, dx + B(x, y)\, dy$]

QUESTION 9. a) Given the force field $F(x, y, z) = (2x + z)i + 2yj + (2z + x)k$. Is $F(x, y, z)$ conservative? If yes, find a function $g(x, y, z)$ such that $\nabla g(x, y, z) = F(x, y, z)$.

b) A particle moves along line segments from (0, 0, 3) to (1, 0, 3) to (4,0, 6) to (0, 0, 6). Find the work done by the above force $F(x, y, z)$ in moving the particle along the given line segments from (0, 0, 3) to (0, 0, 6).

QUESTION 10. a) Find $\int\int_S curl(F)\ .\ dS$ where S is the part of $z = 10-(x^2+y^2)$ that lies above the plane $z = 6$ and $F(x, y, z) = -2yzi + 12xj + 3yk$ (oriented upward).

b) Let F as in (a) but S be the part of of the upper half of the sphere $x^2 + y^2 + z^2 = 40$ that lies above the plane $z = 6$ (oriented upward). Find $\int\int_S curl(F)\ .\ dS$. Just write down your answer!! No WORK IS NEEDED HERE. [Hint: Just bend your head and show some respect to Stoke's Theorem]

QUESTION 11. a) Find the volume of the part of the upper half sphere $x^2 + y^2 + z^2 = 16$ that lies below the cone $z = \sqrt{3x^2 + 3y^2}$. Use triple integrals and spherical coordinates. JUST WRITE DOWN THE INTEGRALS AND DO NOT EVALUATE!!.

b) Find the volume of the solid that is enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the (upper half) sphere $x^2 + y^2 + z^2 = 18$. Use triple integrals and cylindrical coordinates. JUST WRITE DOWN THE INTEGRALS AND DO NOT EVALUATE!!.

QUESTION 12. a) Find the volume of the solid object in 3D that has a basis D, where D is the region in the first quadrant of the xy-plane that is enclosed by the y-axis, the line y = 4 and the line y = x - 1. The height of the solid object is determined by $z = e^{0.5y^2+y+1}$.

b) Consider the curve (in the xy-plane) $C : x = 2$ where $0 \le y \le 4$. Find the area of the side that is between $z = 2e^{2y+x^2+1}$ and the curve C.

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